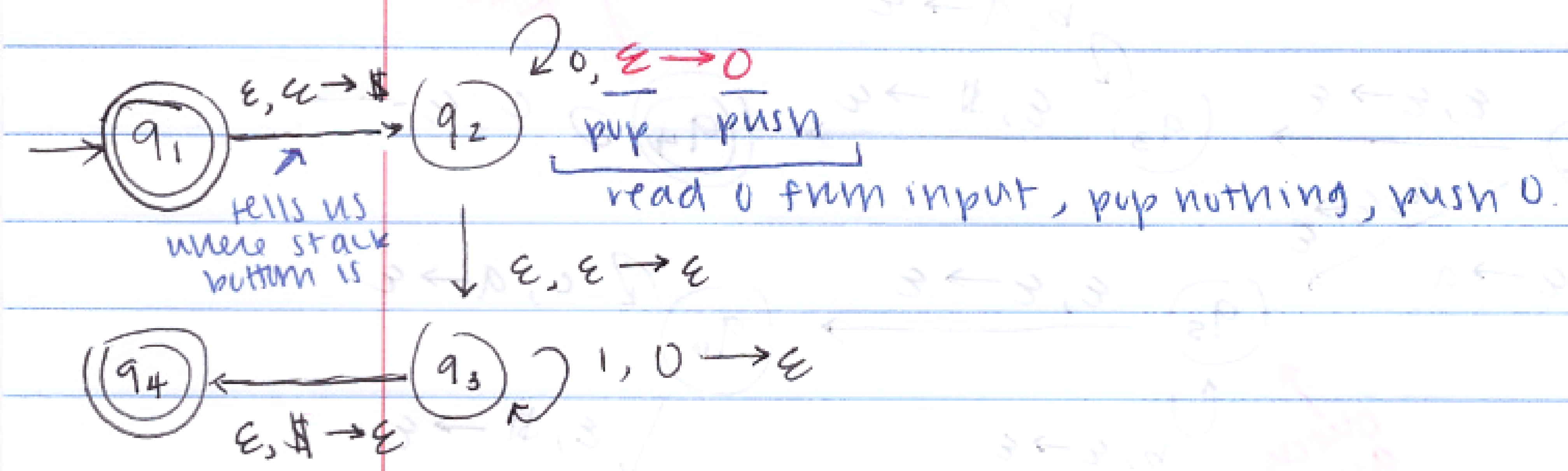
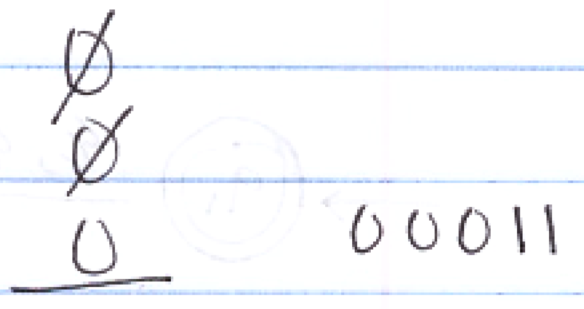
MIDTERM ON OCTOBER 24th!

SEPTEMBER 28, 2017.
Class Notes

Pushdown Automata (PDA)

PDA is NFA with stack. recall $L = \{0^n 1^n \mid n \geq 0\}$.





Formal Definition of PDA.

A PDA is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$ s.t.

① Q is a finite set of states   recall: $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$

② Σ is input alphabet

③ Γ is stack alphabet

④ $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \longmapsto P(Q \times \Gamma_\varepsilon)$ power set

⑤ $q_0 \in Q$ is start state

⑥ $F \subseteq Q$ is set of accept states

PDA L

$Q = \{q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$, $\Gamma = \{0, \$\}$

$F = \{q_1, q_4\}$

δ given by:

input:	0			1			ε		
pop off stack:	0	\$	ε	0	\$	ε	0	\$	ε
q_1									$\{(q_2, \$)\}$
q_2			$\{(q_2, 0)\}$						$\{(q_3, \varepsilon)\}$
q_3				$\{(q_3, \varepsilon)\}$				$\{(q_4, \varepsilon)\}$	
q_4									

Formal definition of acceptance

M accepts input w if w can be written $w = w_1 w_2 \cdots w_m$ for $m \geq 0$ where $w_i \in \Sigma_\varepsilon$, and $\exists$ sequences of states $(r_0, r_1, \ldots, r_m)$ and $\exists$ strings $s_0, s_1, \ldots, s_m \in \Gamma^*$ s.t.

① $r_0 = q_0$ and $s_0 = \varepsilon$

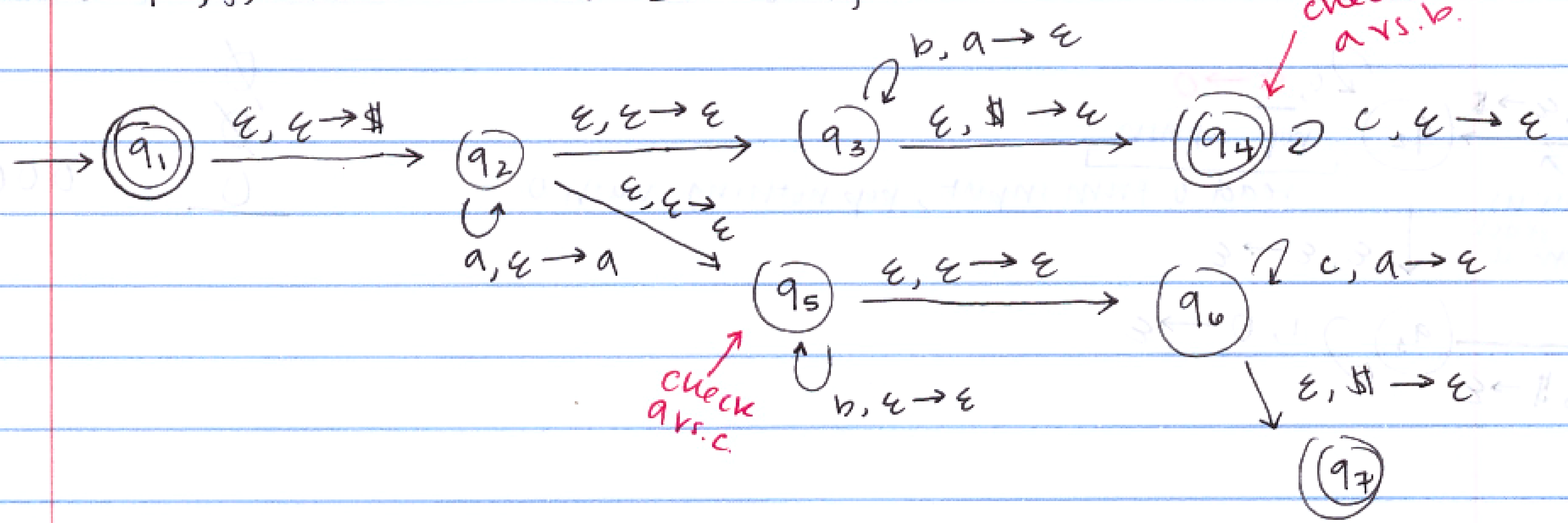
② For $i = 0, \ldots, m-1$, $(r_{i+1}, b) \in \delta(r_i, w_{i+1}, a)$ where $s_i = at$ and $s_{i+1} = bt$ for some $a, b \in \Gamma_\varepsilon$ and $t \in \Gamma^*$

③ $r_m \in F$

example 2.16

Example: of PDAs

$L = \{a^i b^j c^k \mid i,j,k \geq 0 \text{ and } (i=j \text{ or } i=k)\}$



~~a~~
~~a~~
~~\$~~

even baab

odd batab t

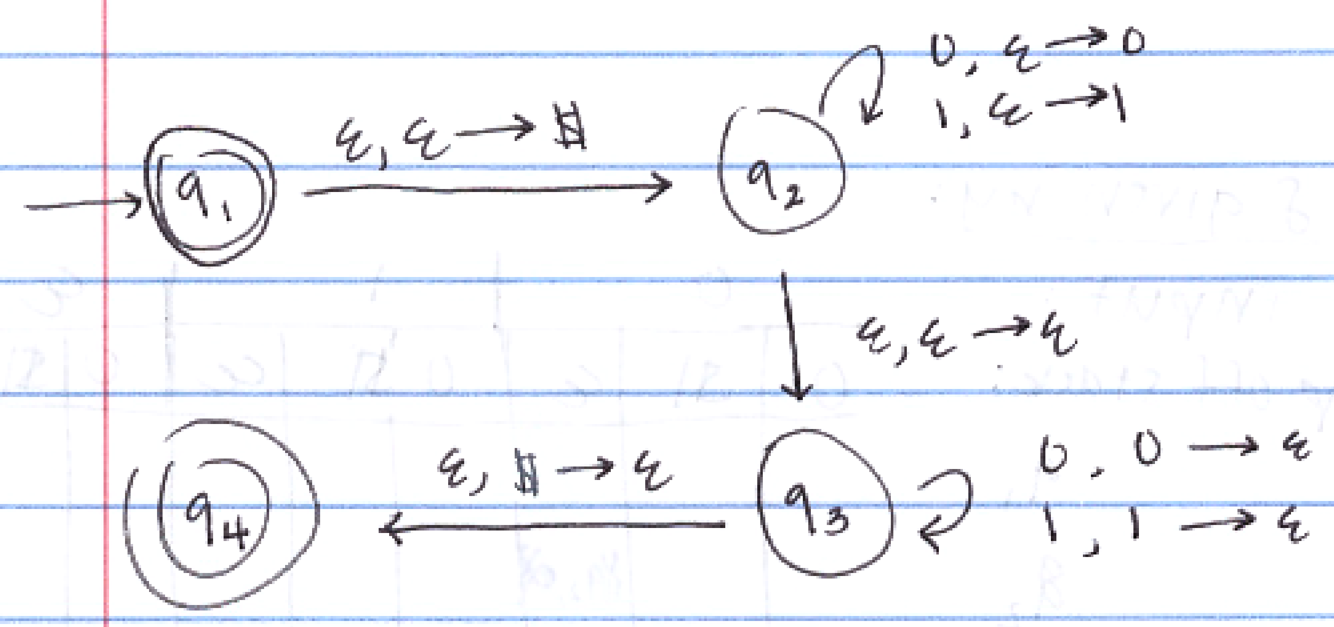
~~a~~ ~~a~~ ~~b~~ ~~b~~ ~~c~~

2.18

Example:

$L = \{ww^R \mid w \in \{0,1\}^*\}$ where w^R is reverse of w.

w = car
w^R = rac



Theorem 2.20 A language is context-free iff some PDA recognizes it